STUDYING THE FREE CONVECTION IN CLOSED AXISYMMETRIC SPACES

Yu. A. Kirichenko, V. N Shchelkunov, and P. S. Chernyakov

UDC 536.25

We have developed an analytical method of calculating the heat-transfer coefficients, the boundary-layer thicknesses, and the temperature and velocity distributions in the boundary layer for the quasisteady free convection in closed axisymmetric spaces for a given heat-flux density at the boundary. We have derived experimental relationships for the heat-transfer coefficient in the case of free convection in a sphere.

Very little research has been done on the free convection in closed spaces at whose surfaces the heat-flux density is known. In [1] we find an investigation of convection in a sphere for a laminar regime, with the sphere heated by steam; in [2] the study deals with convection between two spheres.

Let us examine a closed axisymmetric vessel filled with an incompressible viscous fluid exhibiting an initial temperature T_0. Let the heat-flux density q be specified at the vessel surface S for $t > 0$; x and y are, respectively, the longitudinal and transverse coordinates, associated with the surface of the vessel.

We assume that the following conditions are satisfied:

a) the flow of the liquid is quasisteady, laminar, axisymmetric, or plane;

b) the Grashof number is substantially greater than unity and the Prandtl number is on the order of unity;

c) the entire area occupied by the liquid can be divided into two subregions: the boundary layer of thickness δ and the main core;

d) the liquid flow in the core is ideal;

e) the temperature of the core is equal to the mean-volume temperature of the liquid;

f) the thickness of the thermal boundary layer is equal to the thickness of the dynamic boundary layer;

g) the thickness of the boundary layer is a constant quantity;

h) the thermophysical properties of the liquid are independent of temperature.

We will seek the liquid temperature T and the liquid velocity v in the following form:

$$\begin{gathered} T = T_0 + \gamma t + \tau(X, Y, Z), \\ \mathbf{v} = \mathbf{v}(X, Y, Z), \end{gathered} \tag{1}$$

where X, Y, and Z are Cartesian coordinates; $\gamma = Q/V$.

Having substituted (1) into the equation for nonsteady free convection [3] and using hypotheses a) through g), as well as boundary-layer theory [4], we derive the following equations:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \Pr \frac{\partial^2 u}{\partial y^2} + \mathrm{Gr}\,\Pr^2 \tau \Phi(x), \tag{2'}$$

Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR, Khar'kov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 16, No. 6, pp. 977-983, June, 1969. Original article submitted July 19, 1968.

$$u\frac{\partial\tau}{\partial x}+v\frac{\partial\tau}{\partial y}=\frac{\partial^2\tau}{\partial y^2}-\gamma, \tag{2''}$$

$$\frac{\partial}{\partial x}\left(ur_0(x)\right)+r_0(x)\frac{\partial v}{\partial y}=0 \tag{2'''}$$

and the boundary conditions

$$\begin{gathered} u|_{y=0}=0,\ v|_{y=0}=0,\ u|_{y=\delta}=f,\ \left.\frac{\partial u}{\partial y}\right|_{y=\delta}=0,\\ \tau|_{y=\delta}=0,\ \left.\frac{\partial\tau}{\partial y}\right|_{y=\delta}=0,\ \left.\frac{\partial\tau}{\partial y}\right|_{y=0}=-q(x). \end{gathered} \tag{3}$$

Here it is assumed that the heat from the walls is transported to the boundary layer and we then have a transfer of mass and heat from the boundary layer to the core.

We will use the method of integral relationships [4] to solve problem (2)-(3).

The profile for the temperature τ will be sought in the following form:

$$\tau=\tau_0+\tau_1\frac{y}{\delta}+\tau_2\left(\frac{y}{\delta}\right)^2+\tau_3\left(\frac{y}{\delta}\right)^3+\tau_4\left(\frac{y}{\delta}\right)^4. \tag{4}$$

We find the coefficients τ_0, τ_1, τ_2, τ_3, and τ_4 from the following conditions:

$$\begin{gathered} \tau|_{y=\delta}=0,\ \left.\frac{\partial\tau}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial^2\tau}{\partial y^2}\right|_{y=\delta}=\gamma,\\ \left.\frac{\partial\tau}{\partial y}\right|_{y=0}=-q(x),\ \left.\frac{\partial^2\tau}{\partial y^2}\right|_{y=0}=\gamma. \end{gathered} \tag{5}$$

Conditions $\left.\frac{\partial^2\tau}{\partial y^2}\right|_{y=\delta}=\gamma$, $\left.\frac{\partial^2\tau}{\partial y^2}\right|_{y=0}=\gamma$ are found by substitution of (3) into (2").

Substituting (4) into (5), we find

$$\begin{gathered} \tau_0=\frac{\delta}{2}q,\ \tau_1=-q\delta,\ \tau_2=\frac{\gamma\delta^2}{2},\\ \tau_3=\delta(q-\gamma\delta),\ \tau_4=\frac{\delta}{2}(\gamma\delta-q). \end{gathered} \tag{6}$$

We seek the profile of the longitudinal velocity component in the form

$$u=A_0+A_2\left(\frac{y}{\delta}\right)^1+f_1\left(\frac{y}{\delta}\right)^2+A_3\left(\frac{y}{\delta}\right)^3. \tag{7}$$

We find the coefficients A_0, f_1, A_2, and A_3 from the conditions

$$\begin{gathered} u|_{y=\delta}=f,\ u|_{y=0}=0,\ \left.\frac{\partial u}{\partial y}\right|_{y=\delta}=0,\\ \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0}=-\frac{\mathrm{Gr}\,\mathrm{Pr}\,\Phi\delta q}{2}. \end{gathered} \tag{8}$$

The condition $\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0}=-\frac{\mathrm{Gr}\,\mathrm{Pr}\,\Phi q\delta}{2}$ is found by substituting (3) into (2').

Substituting (7) into (8) yields

$$\begin{gathered} A_2=\frac{3}{2}f-\frac{1}{2}f_1,\ f_1=-\frac{1}{4}\mathrm{Gr}\,\mathrm{Pr}\,\delta^3 q\Phi,\ A_3=-\frac{1}{2}A_1-\frac{1}{2}f,\\ u=\frac{1}{2}\left[f\left(3\frac{y}{\delta}-\left(\frac{y}{\delta}\right)^3\right)+f_1\left(2\left(\frac{y}{\delta}\right)^2-\left(\frac{y}{\delta}\right)^3-\frac{y}{\delta}\right)\right]. \end{gathered} \tag{9}$$

TABLE 1. Values of the Integral Heat-Transfer Coefficients and the Thicknesses of the Thermal Boundary Layer as Functions of the Grashof and Prandtl Numbers

Pr	Ra							
	10^7		10^8		10^9		10^{11}	
	δ	Nu	δ	Nu	δ	Nu	δ	Nu
1	0,0847	23,61	0,0549	36,41	0,0352	56,75	0,0143	140,14
2	0,0846	23,64	0,0549	36,45	0,0352	56,80	0,0143	140,26
3	0,0846	23,64	0,0548	36,46	0,0352	56,82	0,0143	140,30
4	0,0846	23,65	0,0548	36,47	0,0352	56,84	0,0143	140,30

TABLE 2. Value of the Longitudinal Velocity Component for the Core at the Edge of the Thermal Boundary Layer

Ra	10^7	10^8	10^9	10^{11}
f	324,5	855,1	2176	13360

We integrate (2') and (2") with respect to y from 0 to δ, using hypothesis h) and, having substituted into the derived equations (4) and (9), we obtain a system of two algebraic equations for δ and f.

We calculate the dimensionless integral coefficient of heat transfer on the basis of the formula

$$\mathrm{Nu} = \frac{\left.\dfrac{\partial\tau}{\partial y}\right|_{y=0}}{\left.\tau\right|_{y=0}}. \tag{10}$$

Substituting (4) into (10), we obtain

$$\mathrm{Nu} = \frac{2}{\delta}. \tag{11}$$

On the basis of hypothesis g), in this case all of the dimensionless local heat-transfer coefficients are equal to the dimensionless integral heat-transfer coefficient.

Let us examine a special case of this problem – the quasisteady free thermal convection in a sphere at whose surface a constant heat-flux density is specified.

In this case

$$q(x) = 1,\ r_0(x) = \sin x,\ \Phi(x) = \sin x,\ \gamma = 3,\ \mathrm{Ra} = -\mathrm{Gr}\,\mathrm{Pr}.$$

The equations for δ and f in this case have the form

$$f = \frac{P_1}{P_2}, \tag{12'}$$

$$-0.557143\,\delta^2 P_1(\delta) + P_1(\delta)P_2(\delta)[6\,\mathrm{Pr} - 0.535715\cdot10^{-2}\,\mathrm{Ra}\,\delta^5]$$
$$-\frac{\mathrm{Ra}\,\mathrm{Pr}\,\delta^8}{1680}P_2^2(\delta) + \frac{\pi}{20}\mathrm{Ra}\,\mathrm{Pr}\,\delta^4 P^2(\delta) + \frac{\pi}{40}\mathrm{Ra}\,\mathrm{Pr}\,P_2^2(\delta)\,\delta^3 = 0, \tag{12"}$$

where

$$P_1(\delta) = -0.1339285\cdot10^{-2}\,\mathrm{Ra}\,\delta^6 + 0.017113095\,\mathrm{Ra}\,\delta^5 - 2 + 6\delta;$$
$$P_2(\delta) = \delta^2[1.15535715 - 0.06607143\delta].$$

The roots of Eq. (12") were calculated on the M-20 computer in accordance with the Mueller method [5] Ra = 10^7-10^{11} and Pr = 1-5. As a result of the calculations we found that (12") has eight complex roots and four real roots, one of which is less than unity. This smallest real root of (12") was taken as the thickness of the thermal boundary layer. The results from the calculation of the thermal boundary-layer thickness δ and for Nu are shown in Table 1.

We see from Table 1 that the heat-transfer coefficient is virtually an exclusive function of the Rayleigh number. Using the data of Table 1, we can demonstrate that a linear relationship exists between ln Nu and ln Ra. As a result of the processing of these data, we find that

$$\mathrm{Nu} = 1.044\,\mathrm{Ra}^{0.1933}, \tag{13'}$$
$$\delta = 1.92\,\mathrm{Ra}^{-0.1933}. \tag{13"}$$

TABLE 3. Temperature Distribution in the Thermal Boundary Layer and the Values of the Heat-Transfer Coefficients

x^0	0	22,5	45	67,5	112,5	135	157,5	180
Ethyl alcohol, Ra=1,8·10¹¹; R=0,15 m; q=8,4·10² W/m²								
$\Delta\overline{T}'$deg	4,2	4,6	4,1	3,1	3,8	4,5	3,5	3,9
$\Delta\overline{T}''$deg	4,7	4,8	4,7	4,9	5	4,5	3,8	4,6
$\Delta\overline{T}'''$deg	4,7	4,8	4,7	4,9	5	5	5,3	6,0
$\alpha\cdot10^{-2}$	1,8	1,7	1,8	1,7	1,7	1,8	1,6	1,4
$\overline{\alpha}\cdot10^{-2}$				1,7				
Distilled water Ra=7,2·10⁹; R=0,15 m; q=3,4·10² W/m²								
$\Delta\overline{T}'$deg	1,2	0,7	1,3	0,8	0,8	1,0	1,1	0,9
$\Delta\overline{T}''$deg	1,3	1,2	1,3	1,2	1,2	1,2	1,3	1,4
$\Delta\overline{T}'''$deg	1,3	1,2	1,4	1,3	1,2	1,4	1,5	1,7
$\alpha\cdot10^{-2}$	2,6	2,8	2,6	2,8	2,8	2,4	2,3	2,0
$\overline{\alpha}\cdot10^{-2}$				2,58				

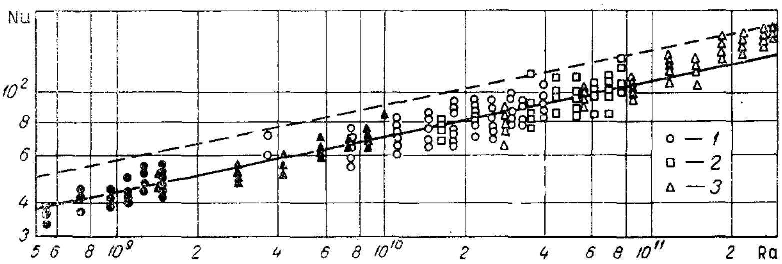

Fig. 1. The dimensionless heat-transfer coefficient as a function of the Ra number: 1, 2) water, $\overline{T}$ = 30 and 60°, respectively, with a vessel whose diameter is 0.3 m; 3) ethyl alcohol, $\overline{T}$ = 30°, in a vessel whose diameter is 0.3 m; the filled dots correspond to the data derived for a vessel with a diameter of 0.15 m.

Table 2 shows the values of f as functions of Ra.

Let us examine the second special case – the quasisteady free convection in an infinitely long horizontal cylinder at whose surface a constant heat-flux density is specified. In this case

$$r_0(x) = 1, \quad \Phi(x) = \sin x, \quad q = 1, \quad \gamma = 2$$

and the equation for δ.

$$0.017113095\,\mathrm{Ra}\,\delta^5 + 4\pi\delta - 2\pi = 0. \tag{14}$$

The roots of Eq. (14) were calculated by the Mueller method on an M-20 computer for Ra = 10^7-10^{11} Equation (14) has four complex roots and one real root; $\ln\delta$ is a linear function of $\ln\mathrm{Ra}$:

$$\mathrm{Nu} = 0.711\,\mathrm{Ra}^{0.1943}, \tag{15'}$$

$$\delta = 2.81\,\mathrm{Ra}^{-0.1943}. \tag{15''}$$

We undertook an experimental investigation of the free convection in a spherical space completely filled with a liquid, using two spherical vessels having diameters of 0.15 and 0.3 m, with boundary conditions of the second kind. The experimental method is described in [6]. The studies were performed with distilled water at an initial temperature of 20 and 50° and on ethyl alcohol (with a concentration of 96%) and an initial temperature of 20°, with heat-flux densities ranging from $1.7\cdot10^2$ W/m² to $1.8\cdot10^3$ W/m², in a Rayleigh number range of $1.5\cdot10^8 \leq \mathrm{Ra} \leq 3.2\cdot10^{11}$.

The investigations revealed that the temperature distribution is symmetrical with respect to the vertical axis and that the heat-transfer process at the "wall–liquid" boundary at a constant heat-flux density is quasisteady in nature, since on elapse of a brief time interval following the onset of the process a constant time-averaged temperature difference is established in the thermal boundary layer for each point of the surface of the sphere and the heating curves for the corresponding points of the liquid and the shell, situated at the edge of the boundary layer, represent parallel straight lines.*

The maximum time interval in which the boundary layer is formed corresponded in our tests to a Fourier number equal to $4 \cdot 10^{-4}$. In the ideal case, i.e., in the absence of heat resistance on the part of the shell and with no heater inertia, the indicated time will evidently be even smaller.

Analysis of the temperature distribution through the thickness of the thermal boundary layer for various regions of the boundary shows that the intensity of the heat transfer over the greater portion of the "wall–liquid" boundary is identical, since we observe no significant difference, whether in the thickness of the boundary layer (Table 3), or in the magnitude of the temperature difference through the layer. A slight exception is represented by the upper region of the sphere, where the value of the heat-transfer coefficient is reduced by 10-15% for x = 160° and by 15-20% for x = 180°. The reduction in the intensity of the heat transfer in the upper region of the space can be explained by the effect of surface-heating orientation.

Since the reduction in the heat-transfer coefficient in the upper region (160° ≤ x ≤ 180°) is insignificant and since the region itself is small, we can describe the process of heat transfer at the boundary of the space by an integral heat-transfer coefficient defined as follows:

$$\bar{\alpha} = \frac{1}{n} \sum_{i=1}^{n} \frac{q}{\Delta \bar{T}_i (x)}, \tag{16}$$

where $\Delta\bar{T}_i$ is the time-averaged temperature difference across the boundary layer for a given angle x; x is the coordinate angle in the spherical system of coordinates; n is the number of measurement point.

Table 3 shows the values for the temperature difference across the boundary layer for various regions of the boundary, and for the local and integral heat-transfer coefficients for two experiments. Here $\Delta\bar{T}'$, $\Delta\bar{T}''$, and $\Delta\bar{T}'''$ are the values of the temperature differences across the boundary layer at distances of 1, 2, and 5 mm from the shell, respectively.

The function for the integral heat-transfer coefficient in the Rayleigh number interval $3.5 \cdot 10^8 < \mathrm{Ra} < 10^{11}$ shown in generalized coordinates in Fig. 1 has the form

$$\mathrm{Nu} = A\,\mathrm{Ra}^k, \tag{17}$$

where A = 0.7 ± 0.1 and k = 0.2 ± 0.01.

Function (17) was obtained on the M-20 computer by the method of least squares, after processing more than 400 experimental points. The experimental points on the curve for $\mathrm{Ra} > 10^{11}$ were not taken into consideration, because of the possible change in the heat-transfer regime at Rayleigh number values in excess of 10^{11}.

The dashed line in Fig. 1 shows the plotting of (13').

Comparison of the values of the heat-transfer coefficients calculated from (13') and (17) shows that the assumptions adopted in the development of the theoretical model are valid for the greater portion of the volume. Because the theoretical relationship was derived for an entire class of liquids characterized by a Pr number on the order of unity, function (17) can also be extended to the entire class of similar fluids for a Rayleigh number range from 10^7 to 10^{11}.

NOTATION

- T is the temperature;
- $\bar{T}$ is the average-volume temperature;
- a is the coefficient of thermal diffusivity;
- ν is the coefficient of kinematic viscosity;
- β is the coefficient of volume expansion;

*The time-averaged temperature difference in this case is understood to refer to the difference averaged over the time interval substantially smaller than the time required to perform the test. The need to speak of a time-averaged temperature difference for the thermal boundary layer results from the fluctuations in temperature noted within the limits of this layer [6].

g is the acceleration of the force of gravity;
R is a characteristic linear dimension;
q is the heat-flux density;
S is the surface area;
V is the vessel volume;
v is the velocity;
p is the pressure;
$u = v_x$, $v = v_y$;
ρ is the density;
α is the heat-transfer coefficient;
$\bar{\alpha}$ is the integral heat-transfer coefficient;
f is the value of the projection of the longitudinal velocity component of the core onto the x-axis at the edge of the boundary layer;
$\Phi(x)$ is the projection of the acceleration of the force of gravity onto the x-axis;
$r_0(x)$ is the radius of curvature for the meridional cross section of the vessel;
t is the Fourier number;
$Nu = \bar{\alpha}R/\lambda$ is the integral heat-transfer coefficient (the Nusselt number);
$Gr = g\beta R^4 q/\nu^2\lambda$ is the modified Grashof number;
Pr is the Prandtl number;
$Ra = GrPr$;
$Q = \int_s q ds$;
h is the thickness of the thermal boundary layer;
$\delta = h/R$;
x, y are, respectively, the longitudinal and transverse coordinates in the coordinate system associated with the surface of the vessel;
X, Y, Z are Cartesian coordinates.

All of the quantities in the theoretical portion of this paper are dimensionless.

LITERATURE CITED

1. E. Schmidt, Chemie-Ingenieur-Technik, 28, No. 3, 175 (1956).
2. E. H. Bishop, L. R. Mack, and I. A. Scanlan, Journal Heat and Mass Transfer, 9, No. 7, 649 (1966).
3. L. D. Landau and E. M. Lifshits, The Mechanics of Continuous Media [in Russian], Gostekhizdat (1953).
4. S. S. Kutateladze, Fundamentals of the Theory of Heat Transfer [in Russian], Mashgiz (1957).
5. I. S. Berezin and N. S. Zhidkov, Methods of Calculation [in Russian], Vol. II, Fizmatgiz (1959).
6. Yu. A. Kirichenko, V. N. Shchelkunov, and S. A. Komarova, "Investigating the transfer of heat in a spherical volume," Heat and Mass Transfer [in Russian], Vol. I (1968).